BIT*: Batch Informed Trees for Optimal Sampling-based Planning via Dynamic Programming on Implicit Random Geometric Graphs

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Abstract—Path planning in continuous spaces has traditionally been divided between discrete and sampling-based techniques. Discrete techniques use the principles of dynamic programming to solve a discretized approximation of the problem, while sampling-based techniques use random samples to perform a stochastic search on the continuous state space.

In this paper, we use the fact that stochastic planners can be viewed as a search of an implicit random geometric graph (RGG) to propose a general class of planners named Bellman random trees (BRTs) and derive an anytime optimal sampling-based algorithm, batch informed trees (BIT*).

BIT* searches for a solution to the continuous planning problem by efficiently building and searching a series of implicit RGGs in a principled manner. In doing so, it strikes a balance between the advantages of discrete methods and sampling-based planners. By using the implicit RGG representation, defined as a set of random samples and successor function, BIT* is able to scale more effectively to high dimensions than other optimal sampling-based planners. By using heuristics and intelligently reusing existing connections, like discrete lifelong planning algorithms, BIT* is able to focus its search in a principled and efficient manner.

In simulations, we show that BIT* consistently outperforms optimal RRT (RRT*), informed RRT* and fast marching trees (FMT*) on random-world problems in $\mathbb{R}^2$ and $\mathbb{R}^3$. We also present preliminary theoretical analysis demonstrating that BIT* is probabilistically complete and asymptotically optimal and conjecture that it may be optimally efficient under some probabilistic conditions.

I. INTRODUCTION

Path planning techniques for robotics are roughly divided into two categories: graph-based searches that operate on a discrete representation of the problem and stochastic incremental searches that grow graphs by randomly sampling the continuous state space.

Graph-based searches, such as Dijkstra’s algorithm [1] and A* [2], use dynamic programming [3], [4] to solve a discrete representation of the problem exactly. These algorithms are not only resolution complete but also resolution optimal, always finding the optimal solution to the discretization of the planning problem, if one exists. A* accomplishes this efficiently by using a heuristic, $f(\cdot)$, to estimate the potential cost of a solution constrained to pass through a state. The result is an optimally efficient algorithm, meaning that any other optimal algorithm with the same heuristic will expand at least as many states as A* [2].

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The original A* algorithm returns only the final optimal solution to a given problem. Anytime algorithms based on A* exist that initially provide suboptimal solutions quickly while continuing to calculate the optimal solution in an efficient manner [5]–[8]. The original A* algorithm is also inefficient at handling changes to the graph; even small changes require a complete recalculation of the solution. Replanning algorithms based on A* are capable of incorporating changes in ways that efficiently reuse the existing solutions, minimizing the new calculations required [8]–[11].

The quality of the continuous-space solution resulting from these graph-search techniques depends heavily on the chosen discretization of the problem. Finer discretization increases the quality of the solution [12], but also increases the computational effort necessary to find it. This becomes a significant problem when planning in high-dimensional spaces, such as for manipulation planning, as the size of the discrete state space grows exponentially with the number of dimensions, a problem Richard Bellman himself referred to as the \textit{curse of dimensionality} [13].

Despite these limitations, dynamic programming has long been applied to path-planning algorithms [14] including for nonholonomic robots [15], [16], kinodynamic planning [17], [18] and even manipulation planning [19]. Most techniques used regularly-spaced discretizations, but existing work also includes variable resolutions [20] or randomized graphs [21].

Sampling-based planners, such as probabilistic roadmaps (PRM) [23], rapidly-exploring random trees (RRT) [24], and expansive-space trees (EST) [25], [26], avoid the problems
of discretization by representing the planning domain as a set of samples drawn randomly from a continuous state space. This allows them to scale effectively to high-dimensional configuration spaces. They are **probabilistically complete**, having a probability of finding a solution, if one exists, that goes to unity as the number of samples goes to infinity. Optimal variants, such as RRT*, PRM* [27] and FMT* [28], are also **asymptotically optimal**, having the probability that the found solution is optimal, if one exists, also going to unity as the number of samples goes to infinity. To date, very little has been said about the efficiency with which these algorithms find the optimal solution; however, informed RRT* [22] showed that a heuristic can be used to improve the rate at which a solution converges towards the optimum.

In this paper, we argue that optimal sampling-based planning algorithms that generate trees (e.g., RRT*, FMT*) can be unified as a class of algorithms we refer to as Bellman random trees (BRTs). This class of planners uses (exact or approximate) dynamic programming to build an explicit tree from the implicit random geometric graph (RGG) defined by the random samples. We unify these observations with dynamic programming to develop an anytime optimal planning algorithm, batch informed trees (BIT*) (Figs. 1, 2).

BIT* is a BRT algorithm that combines the anytime nature of incremental BRTs (i.e., RRT*) and the efficiency of bulk BRTs (i.e., FMT*) with a heuristic to focus the search. This not only prioritizes low-cost solutions, but also minimizes the size of the explicit tree constructed from the implicit RGG. It does this by using dynamic programming to solve increasingly dense RGGs that are constructed in a principled manner, allowing for both quick initial solutions as well as a focused search for improvements that reuses existing information. Its performance depends on the properties of dynamic programming and RGGs, making it simple to analyze. It is shown that it is both probabilistically complete and asymptotically optimal, and it is conjectured that under some currently unknown probabilistic conditions, it may also be optimally efficient.

The performance of BIT* is demonstrated relative to existing optimal algorithms through a series of experiments on random worlds in $\mathbb{R}^2$ (Figs. 4, 5) and $\mathbb{R}^8$ (Figs. 6, 7). BIT* more reliably finds solutions and finds solutions of equivalent cost faster than RRT*, informed RRT*, and FMT*.

The remainder of this paper is organized as follows. Section II presents the general BRT class of planners while Section III presents the specific BIT* algorithm. Section IV presents initial analysis of the algorithm. Section V presents the experimental results in more detail and Sections VI and VII provide a discussion of the algorithm and ongoing work and a conclusion.

### II. Bellman Random Trees (BRTs)

Karaman and Frazzoli [27] recognized that the random samples from the state space in a planning algorithm can be modelled as an RGG. RGGs are a probabilistic structure with a number of important statistical properties depending on their specific type. For planning, the type of RGG depends on the connection scheme used by the planner, but common types include $r$-disc graphs [29], [30], $k$-nearest neighbours graphs [31], [32], and online graphs [33].

Karaman and Frazzoli [27] provide excellent definitions of these graphs as they relate to planning. An $r$-disc RGG, $G^\text{near}_r(n, r)$, is defined by a set of $n$ randomly distributed vertices and a radius of connection $r$, with edges existing between vertices if and only if the distance between them is less than $r$. A $k$-nearest neighbours graph, $G^\text{near}_k(n, k)$, is similarly defined by a set of $n$ random vertices but with edges existing between vertices and their $k$-th nearest neighbours. An online nearest-neighbour graph, $G^\text{onn}_n(n)$, is defined by a sequence of $n$ random vertices with edges existing only between a vertex and the preceding vertex in the sequence that minimizes the edge length. These different types of RGGs will be necessary in the following sections to describe how the different algorithms work.

Any set (or sequence) of samples from the state space, $V = \{x \in X_{\text{free}}\}$, can be viewed by an implicit RGG with a specified successor function that uncovers a given vertex’s connections, $\tau : v \in V \rightarrow \{(v, w \in V)\}$. For example, in the implicit $r$-disc graph this successor function would be a nearest-neighbours search.
By not explicitly storing the edge set, implicit graphs are a compression that reduces the size of the graph while increasing the computational effort necessary to search it. Sampling-based planning algorithms that return a tree can therefore be viewed as a procedure that searches an implicit RGG for an explicit spanning tree, with efficient algorithms minimizing computation. The successor function depends on the specific planning algorithm and in some conditions, may also uncover new samples (e.g., dynamic constraints, just-in-time sampling, etc.). Given the common use of approximate or exact dynamic programming to perform this search on the RGG we propose naming this class of planners Bellman random trees (BRTs).

RRT* is an example of an iterative, anytime BRT and can be viewed as a random search through an RGG for an approximate shortest-path spanning tree. Given a sequence of random samples, \( \{x_1, x_2, \ldots, x_n\} \), RRT* incrementally does this by performing an approximate dynamic programming step for the \( i \)-th vertex. Considering only the current spanning tree, the current online nearest-neighbour RGG, \( G_i^{\text{omin}}(n_i) \), and the \( r \)-disc RGG, \( G_i^{\text{near}}(n_i, r_i) \), RRT* finds both the edge that minimizes the cost-to-come of the \( i \)-th vertex in the tree and updates nearby vertices in the tree with better edges from the \( r \)-disc graph. The algorithm is anytime and can be run until a solution of suitable quality is found, but the search is unfocused and finds optimal solutions to every state in the problem. As the order of vertices added to the spanning tree is random, so is its construction and the search for a solution to the optimal planning problem. In finite time, the final solution will depend not only on the random samples but also the order in which they are processed.

FMT* is an example of a bulk BRT and can be viewed as an approximation of Dijkstra’s algorithm on an implicit RGG. It methodically builds an approximate shortest-path spanning tree through an implicit \( r \)-disc RGG given by \( n \) samples and a radius connection, \( r \). It processes a queue of open vertices from the tree, at each iteration expanding the vertex in the queue with the cheapest cost-to-come. It applies the successor function to this vertex to uncover vertices in the implicit RGG not in the tree and adds them to the tree in an approximately optimal manner (see Section 3.1 of [28] for a discussion of a rare sub-optimal condition). The ordered processing of the RGG provides a more efficient conversion of the implicit RGG to an explicit spanning tree; however it also finds optimal solutions to every state in the problem. The algorithm is also not anytime, and if a solution requires further improvement a completely new denser search is required.

BIT* balances the advantages of bulk and iterative BRTs to provide an anytime planning algorithm that is faster than both RRT* and FMT*. It does this in a way that is both principled and efficient by introducing a heuristic and applying the principles of dynamic programming to search the implicit RGG. Each batch is searched in an order that prioritizes both growth towards the goal and the exploration of high-quality paths. Section III describes the algorithm in detail.

III. Batch Informed Trees (BIT*)

The definition of the optimal planning problem is as in [27]. Let \( X = [0,1]^d \) be the configuration space of the planning problem and \( X_{\text{obs}} \subset X \) be the states in collision with obstacles. The permissible set of states is then \( X_{\text{free}} = X \setminus X_{\text{obs}} \). Let \( x_s \in X_{\text{free}} \) be the initial state of the planning problem and \( X_{\text{goal}} \subset X_{\text{free}} \) be the desired final state. Let \( \sigma : [0,1] \rightarrow X \) be a sequence of states (a path) and \( \Sigma \) be the set of all nontrivial paths.

The formal definition of the optimal planning problem is then the search for the path, \( \sigma^* \), that minimizes a given cost function, \( c : \Sigma \rightarrow \mathbb{R}_{\geq 0} \), and connects \( x_s \) to \( x_g \in X_{\text{goal}} \) through free space,

\[
\sigma^* = \arg \min_{\sigma \in \Sigma} \{ c(\sigma) \mid \sigma(0) = x_s, \sigma(1) = x_g, \forall s \in [0,1], \sigma(s) \in X_{\text{free}} \},
\]

where \( \mathbb{R}_{\geq 0} \) is the set of non-negative real numbers.

BIT* employs a batch approach to solving the optimal planning problem that allows a balance between the anytime nature of RRT* and the methodical nature of FMT*. It does this by searching for the shortest path from start to goal through a series of increasingly dense \( r \)-disc RGGs defined by batches of random samples. By processing the vertices in batches of size greater than one, BIT* is able to process the samples in an ordered manner, like FMT*. Processing multiple batches also provides a number of advantages over a single large batch (FMT*) or a large number of individual batches (RRT*). First, it is able to return solutions in an anytime manner and run continuously towards the optimal solution, like RRT*. Second, when solution improvements are found, new batches of samples can be drawn from a smaller subset of the problem, increasing sample density only where it matters.

Informally, BIT* works as follows. An \( r \)-disc RGG, \( G_i^{\text{near}}(n_i, r_i) \), is implicitly constructed from \( n_i = n' \) uniformly-distributed random samples from \( X_{\text{free}} \) and an \( r_i \) that satisfies the requirements for asymptotic optimality (see [27]). A heuristically guided search then builds an explicit shortest-path tree through the RGG until a path between \( x_s \) and \( x_g \) is found. The length of this path then defines a heuristically informed subset of the problem, \( X_{\text{II}} \), that contains all possibly better solutions. A new implicit RGG, \( G_{i+1}^{\text{near}}(n_{i+1}, r_{i+1}) \), is then generated from a batch of \( n_{i+1} = n_i + n' \) random samples uniformly-distributed on \( X_{\text{II}} \) and used to update the existing tree. The radius of connection, \( r_{i+1} \), is once again chosen such that it satisfies the requirements for asymptotic optimality. Each batch of samples not only increases the density of the RGG but is also used to update the shortest-path tree through RRT*-style local rewirings. This process is continued as time allows.

To perform the search in an efficient way, BIT* employs a heuristic to guide the search through the RGGs. Not only does this focus the search towards the goal and minimize the number of vertices expanded, but it also prioritizes the...
The function $g_T(x)$ represents the cost-to-come to a state $x \in X$ from the start vertex given the current tree, $T$. We assume that a state not in the tree has a cost-to-come of infinity, i.e., $\forall x \notin V, g_T(x) = \infty$. Given the asymptotic convergence of the RGG representation, these two functions will always bound the unknown true optimal cost to a state from the start, $g(\cdot)$, i.e., $\forall x \in X, \hat{g}(x) \leq g_T(x) \leq g(x)$.

The functions $\hat{c}(x,y)$ and $c(x,y)$ represent an admissible estimate of the cost of an edge and the true cost of an edge between states $x, y \in X$, respectively. We assume that edges that intersect the obstacle set have a cost of infinity and otherwise $\forall x, y \in X, \hat{c}(x,y) \equiv c(x,y)$.

The function $\lambda(\cdot)$ represents the volume of a set (i.e., the Lebesgue measure). Finally, we use the notation $X \leftarrow \{x\}$ and $X \leftarrow \{x\}$ to compactly represent the compounding operations $X \leftarrow X \cup \{x\}$ and $X \leftarrow X \setminus \{x\}$, respectively.

### B. Algorithm

Alg. 1 presents the main algorithm, which can be run for as long as desired and returns a tree, $T := (V,E)$, such that if a solution exists, $x_s$ and $x_g$ are connected through the tree such and their path is minimized given the specific RGG. It does this by using three queues: free states, $Q_{\text{free}}$, potential new edges, $Q_{\text{edge}}$, and potential replacement edges that may improve the existing tree, $Q_{\text{rewire}}$. Initially, the vertex set consists of $x_s$, and the free queue consists of $x_g$ (or samples from $X_{\text{goal}}$) (Alg. 1 Lines 1–3).

For each batch, the queue of edges, $Q_{\text{edge}}$, is populated with all possible connections from the existing tree to states in the free queue that could provide a better solution (Alg. 1 Lines 5–6). This queue is then iteratively processed (Alg. 1 Lines 7–19) to find the minimum path from $x_s$ to $x_g$ through the current RGG that is defined by the samples in $V \cup Q_{\text{free}}$ and the appropriate $r$ given by [27].

At each iteration, we remove and process the potential edge, $(v_m,x_m)$, from $Q_{\text{edge}}$ with the lowest heuristic estimate of a solution for our current graph (Alg. 1 Lines 8–9). This is the pair that minimizes the sum of the cost-to-come through the tree, $g_T(v_m)$, the heuristic estimate of the edge cost $\hat{c}(v_m,w_m)$ and the heuristic estimate of cost-to-
Algorithm 5: Rewire($Q_{\text{rewire}}$)

1. while $Q_{\text{rewire}} \neq \emptyset$ do
2.   $(u_m, w_m) \leftarrow \arg \min_{(u, w) \in Q_{\text{rewire}}} \{ g_T(u) + \hat{c}(u, w) + \hat{h}(w) \};$
3.   $Q_{\text{rewire}} \leftarrow \{ (u_m, w_m) \};$
4.   if $g_T(u_m) + \hat{c}(u_m, w_m) + \hat{h}(w_m) < g_T(x_g)$ then
5.     $E \leftarrow \{ (v, w_m) \mid (v, w_m) \in E \};$
6.     $E \leftarrow \{ (v, w_m) \};$
7.     UpdateRewireQueue($w_m$);
8.   else
9.     $Q_{\text{rewire}} \leftarrow \emptyset$;

If the minimum such edge could not improve our current solution to the goal, $g_T(x_g)$, (Alg. 1 Line 10) then by construction no other edge in $Q_{\text{edge}}$ can and we clear the queue to generate a denser RGG (Alg. 1 Lines 17–18). If the minimum edge could provide a better solution, we evaluate the true cost of the edge (i.e., check the edge for collisions, Alg. 1 Line 11) and if the resulting path can still provide an improvement, we add it to the tree (Alg. 1 Line 12). We then remove the recently added state from the list of free states, $Q_{\text{free}}$, (Alg. 1 Line 13), update the queues with potential edges originating from the newly added state (Alg. 1 Lines 14–15) and process the rewirings (Alg. 1 Line 16).

For problems seeking to minimize path length, the processing order assures that rewirings will not be necessary when processing the first batch of samples.

The subfunctions of the main algorithm are described below:

UpdateEdgeQueue (Alg. 2): Given a vertex in the tree, $v \in V$, the function UpdateEdgeQueue ($v$) updates the edge queue, $Q_{\text{edge}}$, with all possible connections from the vertex to the nearby free states that could improve the current solution, $\{ (v, x) \mid x \in X_n \cap g_T(v) + \hat{c}(v, x) + \hat{h}(x) < g_T(x_g) \}$, where $X_n = \{ x \in Q_{\text{free}} \mid \|x - v\|_2 \leq r \}$. This does this by first calling UpdateFreeQueue ($v$) (Alg. 2 Line 1) to assure that $Q_{\text{free}}$ is appropriately populated. It then prunes the queue of any edges to the given vertex (Alg. 2 Line 2) and adds all potential edges from the given vertex that could be part of a better solution (Alg. 2 Lines 3–4).

UpdateFreeQueue (Alg. 3): Given a vertex in the tree, $v \in V$, the function UpdateFreeQueue ($v$) assures that the volume sampled by $Q_{\text{free}}$ includes the entire volume of the near neighbourhood of $v$. It does this by incrementally sampling the heuristically informed subset containing the neighbourhood of $v$, $f_{\text{req'd}} := \max_{x \in X_n} \{ \hat{f}(x) \}$, where $X_n = \{ x \in X \mid \|x - v\|_2 \leq r \}$. This is accomplished by sampling prolate hyperspherical shells so that the sampling density of the search subset remains uniform (Fig. 3). Each time the function is called, the heuristic value to be sampled, $f_{\text{sample}}$, is checked against the previously sampled heuristic value, $f_{\text{prev}}$ (Alg. 3 Lines 1–2). If it is less, we calculate the sampling volume and density (Alg. 3 Lines 3–4) and sample it (Alg. 3 Lines 5–6). Note, for problems seeking to minimize path length $f_{\text{req'd}} \leq \hat{f}(v) + 2r$ will always hold.

Sample: Given two heuristic solution costs, $f_{\text{prev}}, f_{\text{sample}} \in \mathbb{R}_{\geq 0}$, and a sampling density, $\rho \in \mathbb{R}_{\geq 0}$, the function Sample($f_{\text{prev}}, f_{\text{sample}}, \rho$) returns independent and identically distributed (i.i.d.) samples from the shell $X_{\text{shell}} := \{ x \in X \mid f_{\text{prev}} < \hat{f}(x) \leq f_{\text{sample}} \}$ such that the number of samples, $q$, results in a sample density $\rho = q/\lambda(X_{\text{shell}})$. For planners seeking to minimize path length, this is equivalent to drawing samples from the prolate hyperspherical shell defined by transverse diameters $f_{\text{prev}}$ and $f_{\text{sample}}$ and foci $x_s$ and $x_g$ [22], [35]. This can be implemented by sampling the larger volume directly and rejecting samples that are inside the smaller volume [22]. Since in high-dimensions the majority of volume in a unit-ball lies near the surface this is relatively efficient, though exact methods are currently sought.

Rewire (Alg. 5): The function Rewire processes the rewiring queue, $Q_{\text{rewire}}$, to minimize the path length through the tree from the start, $x_s$, to every vertex in the tree, $x \in V$. It does this by incrementally removing and processing the potential edge, $(v_m, w_m)$, from $Q_{\text{rewire}}$ with the lowest heuristic solution-cost in our current tree (Alg. 5 Lines 2–3). This is the pair that minimizes the sum of the cost-to-come through the tree, $g_T(v_m)$, the heuristic estimate of the edge cost $\hat{c}(v_m, w_m)$ and the heuristic estimate of cost-to-go, $\hat{h}(w_m)$. If the minimum such edge could not improve our current solution to the goal (Alg. 5 Line 4)

![Fig. 3. An illustration of just-in-time sampling.](image-url)
then by construction no other edge in $Q_{\text{rewire}}$ can and we clear the rewiring queue (Alg. 5 Lines 9–10). If it could, we evaluate the actual cost of the edge, $c(v_m, w_m)$, (i.e., check the edge for collisions, Alg. 5 Line 5), and if it improves the path to $w_m$, replace the existing incoming edge to $w_m$ with $(v_m, w_m)$ (Alg. 5 Lines 6–7).

As discussed in Section VI, in practice we do not check for resulting improvements in the graph, omitting Alg. 5 Line 8. This makes the rewiring algorithm at each batch an approximation as in RRT*, but does not appear to affect the final asymptotic optimality.

IV. ANALYSIS

Without prior knowledge of the planning problem to justify a nonuniform sampling distribution, the best solution we can probabilistically expect from a sampling-based planner is bounded by the properties of RGGs. Similarly, given a specific $G \in G^{\text{rand}}(n, r)$, dynamic programming provides methods to efficiently search the graph for the optimal path from $x_s$ to $x_g$. It naturally follows that the analysis of BIT* will depend on the probabilistic properties of the RGG representation and the method used to search the graphs. Sketches of these proofs are presented below, it is expected that they will be expanded before final submission.

**Theorem 1 (Probabilistic Completeness):** The probability that BIT* returns a solution to a given planning problem, if a solution exists, goes to unity as the number of iterations, $i$, goes to infinity.

**Proof:** (Sketch) For each batch, BIT* searches an $r$-disc RGG that is equivalent to the graph defined by PRM* in [27] for the same samples. Thus, the proof of its probabilistic completeness follows from the probabilistic completeness of PRM* (Theorem 22 in [27]) and the completeness of the graph search performed on the RGG by BIT*.

**Theorem 2 (Asymptotic Optimality):** The probability that the solution returned by BIT* is the optimal solution to a given planning problem, if such a solution exists, goes to unity as the total number of iterations, $i$, goes to infinity.

**Proof:** (Sketch) As in the proof of Theorem 1, we recognize that the $r$-disc RGG searched by BIT* at any batch is equivalent to the graph defined by PRM* for the same samples. Thus, the proof of asymptotic convergence follows from the asymptotic optimality of PRM* (Theorem 34 in [27]) and the ability of BIT* to find the optimal solution in a given graph. The proof that the exact version of BIT* finds the shortest-path tree in a graph follows from the theorems of dynamic programming. The proof that the approximate version (i.e., without cascaded rewiring) of BIT* also finds the shortest-path tree should follow from proofs in [27] and is currently being investigated.

**Conjecture 1 (Probabilistic Optimal Efficiency):** It is conjectured that for a given total number of batches, $b$, number of samples per batch, $n$, a radius of connection in terms of the total number of samples, $r$, and a heuristic, $f(\cdot)$, the exact version of BIT* is the optimally efficient method to solve the planning problem under some probabilistic measure.

**Proof:** (Sketch) For the single batch case, this should follow immediately from the proofs of A* and showing that BIT* is equivalent to A* on an implicit RGG. It is believed that it may be possible to make a more general statement for the general planning problem under some appropriate probabilistic limitations.

V. EXPERIMENTAL RESULTS

BIT* was compared to RRT*, informed RRT* and FMT* with various batch sizes on a series of random worlds in $\mathbb{R}^2$ and $\mathbb{R}^8$ (Fig. 2). The worlds were randomly populated with obstacles (axis-aligned rectangles and hyperrectangles, respectively) such that at most a quarter of the environment was obstructed (Fig. 1). In total, the planners were tested on 12 random worlds (6 in $\mathbb{R}^2$ and 6 in $\mathbb{R}^8$), with each world being solved by the planners 20 different times with different pseudo-random seeds.

In an attempt to minimize the effects of the RRT-steer parameter on RRT* and informed RRT*, we set it equal to the radius given in [27] for asymptotic optimality at each iteration. This is the same calculation used by FMT* and BIT* to define nearest neighbours. We evaluated various samples sizes for FMT* and used batch sizes of $n = 1000$ and $n = 5000$ in $\mathbb{R}^2$ and $n = 5000$ in $\mathbb{R}^8$ for BIT*.

The experiments were run on a MacBook Pro running Ubuntu 12.04 with 4 GB of ram and a 2.67 GHz processor.

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These experiments demonstrated that in low dimensions BIT* finds an initial solution in approximately the same time as other planners, but finds solutions of equivalent cost significantly faster (Fig. 4). This difference is even more pronounced in worlds with narrow passages (Fig. 5). In high dimensions, BIT* significantly outperformed the existing algorithms in terms of time required to find an initial solution, time required to find solutions of equivalent costs (Fig. 6), and also in the probability of finding a solution by a specific time (Fig. 7).

The algorithm implementations were unoptimized and these results are preliminary; however, all algorithms share the same primitive functions which should permit rough relative comparisons. While optimizations will probably have a greater effect on the slower algorithms, since BIT* ultimately performs the fewest operations, the relative order of performance should not change. It would be misleading to present performance versus iteration as each algorithm does a significantly different amount of work per iteration.

VI. DISCUSSION

The BIT* algorithm can be considered to be a search of a series of increasingly-dense $r$-disc RGGs. Thus, after updating the existing tree with rewiring, we generally assume that $g(x) = g_T(x)$ holds for the entire processing of a single $Q_{edge}$ (i.e., a batch). This is equivalent to solving the current RGG while ignoring the future possibility of denser RGGs, which helps limit the size of the queues. In practice, however, we found two situations where the algorithm performed better when we considered the possibility of future rewiring. These are the addition of new edges to the edge queue (Alg. 2, Line 2) and the addition of edges to the explicit tree (Alg. 1, Line 8). At this time, it is not clear if the resulting difference in performance is from a fundamental algorithmic advantage or a limitation in our implementations.

In Section III-A we stated that our heuristic estimate of the edge-cost was always the true edge cost, except for when the edge intersected obstacles, $\hat{c}(\cdot, \cdot) = c(\cdot, \cdot) \cup \infty$. This is an acceptable assumption for planning problems where the calculation of the edge cost is not expensive. For problems where this is not true, the algorithm should be modified to consider whether edges in $Q_{edge}$ could further improve a newly added vertex before they are removed.

To exactly rewire the tree in Alg. 5, all descendents of an updated vertex must also be evaluated for the potential to further improve the tree as shown in Alg. 5, Line 8. Cascading these rewirings can be expensive, and anecdotal evidence suggests that the resulting computational cost outweighs their benefit in iterative algorithms. There exists precedence in both dynamic programming techniques [5] and sampling-based planners [27] to limit this rewiring to only the edges from the new vertex, and as a result we omit Line 8 from Alg. 5 in our implementation of the BIT* algorithm. This makes the graph search performed during each batch an approximate dynamic programming algorithm, though it does not appear to affect the final asymptotic optimality.

In practice, there are a number of possible implementation improvements. The heuristic can be used to remove both samples from $Q_{free}$ and vertices from $V$ that cannot be part of a better solution. In our experiments, we pruned $Q_{free}$ but not $V$. It may also be more efficient to check that the target of a new edge (Alg. 1, Line 8) is not already in the tree than to clear the queue of all such edges each time a new state is added (Alg. 2, Line 2).

VII. CONCLUSIONS & FUTURE WORK

In this paper we define a general class of planners, Bellman random trees (BRTs), that generate an explicit shortest-path tree from an implicit RGG. We use this classification to motivate the development of an optimal anytime planning algorithm, batch informed trees (BIT*). BIT* combines the anytime advantages of RRT* with the ordered-processing advantages of FMT*. It does this by iteratively processing batches of samples, effectively searching a series of increasingly dense RGGs embedded in the continuous configuration space. The sizes of these batches can be viewed as a choice between exploration (small batches) and exploitation (large batches).

It uses a heuristic and dynamic programming to do this in an efficient and principled manner that minimizes changes to the shortest-path tree. This allows it to both quickly find initial solutions and rapidly converge towards the optimal solution. By using a heuristic, it focuses its search towards the goal and minimizes the size of the explicit tree constructed.
from the implicit RGG, thus allowing it to scale more effectively to higher dimensions than existing algorithms. This focus also shrinks the search volume, accelerating the increase in sample density near the optimal solution.

It is shown that the resulting algorithm is probabilistically complete and asymptotically optimal. It is also conjectured that under some unknown probabilistic measure it may be the optimally efficient search for a given choice of batch size, number of batches, connection radius, and heuristic.

BIT* performance was compared to RRT*, informed RRT*, and various sample-densities of FMT* on a series of random worlds in $\mathbb{R}^2$ and $\mathbb{R}^5$. In both state spaces, BIT* outperformed existing algorithms in terms of time required to find solutions of equivalent quality, with the differences becoming more pronounced in higher dimensions and problems with narrow passages.

This manuscript is an early draft of work in preparation for the 2015 IEEE International Conference on Robotics and Automation (ICRA), and as a result there is a significant amount of ongoing work not reflected in this document. This includes experimental, analytical, and work on the document itself. It is clear that the proofs as presented in Section IV are insufficiently thorough, and it is our current focus to improve them as well as replace Conjecture 1 with a proper theorem. It is also our intention to develop and release BIT* in the open motion planning library (OMPL) [36] to facilitate more accurate comparisons and to perform tests on real planning problems (e.g., manipulation planning) as soon as possible.

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REFERENCES


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